Distributed Transient Safety Verification via Robust Control Invariant Sets: A Microgrid Application

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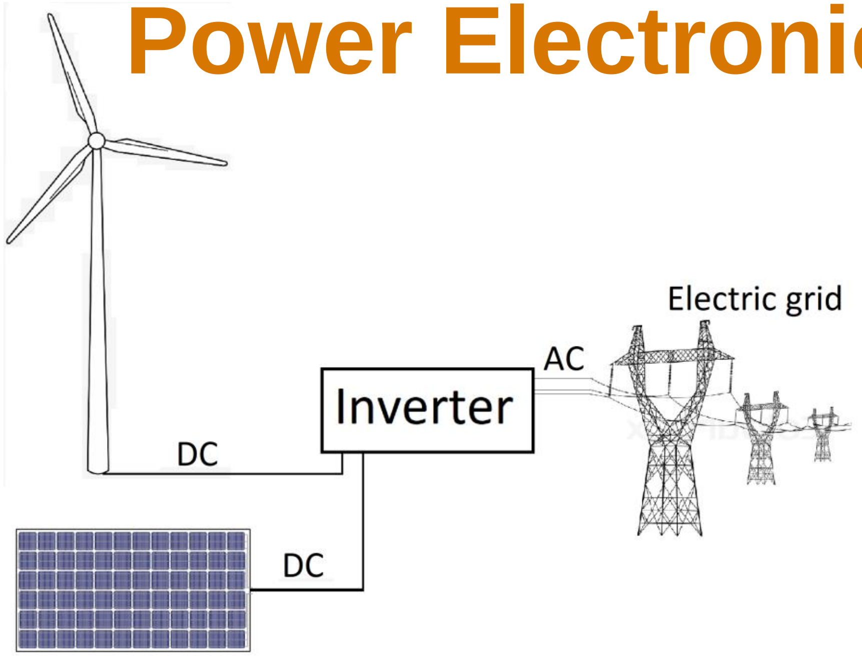
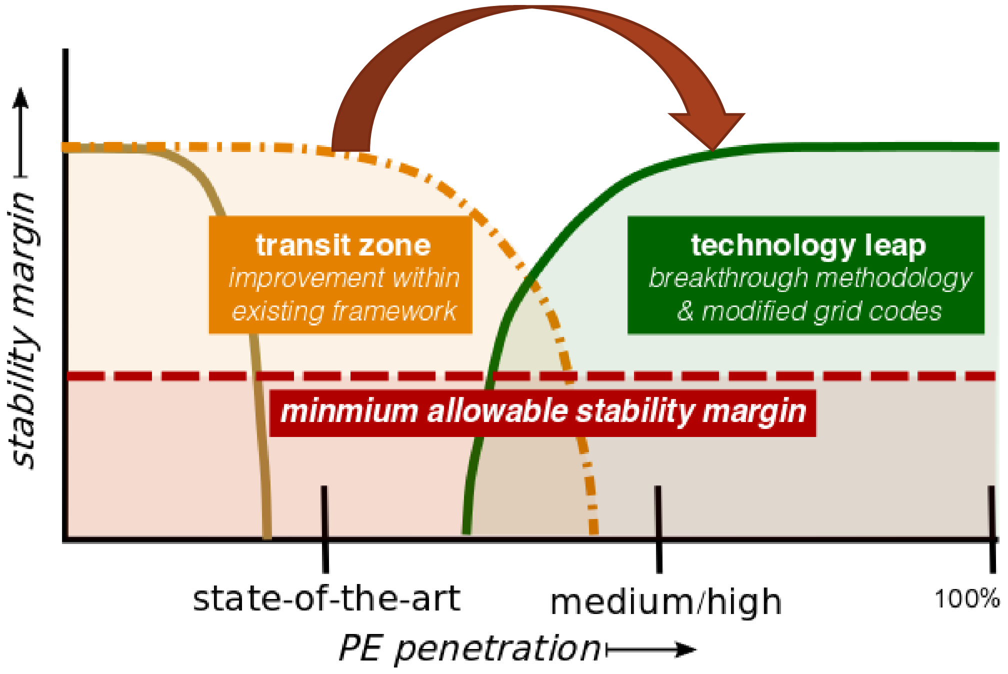
Power Electronics (PE) Interfaced Grid



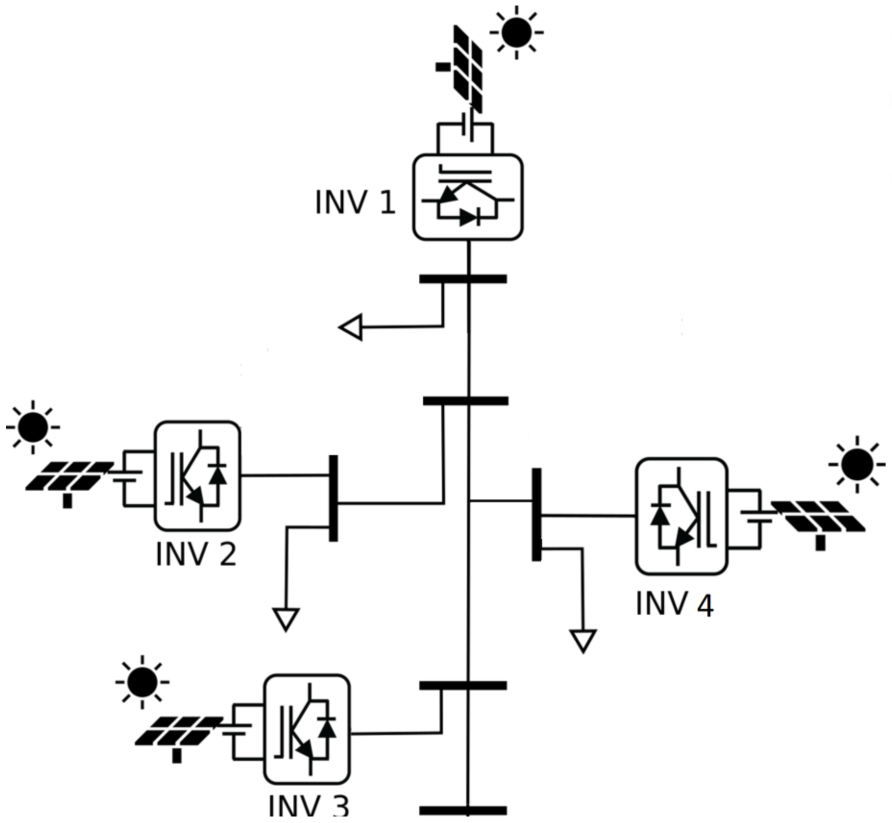
- Existing operational framework is insufficient to deal with the challenges of high PE penetration
  - Lower inertia
  - Larger transients at fast (electromagnetic) timescales
  - Higher uncertainties in power generation
  - Reduced stability and safety margins

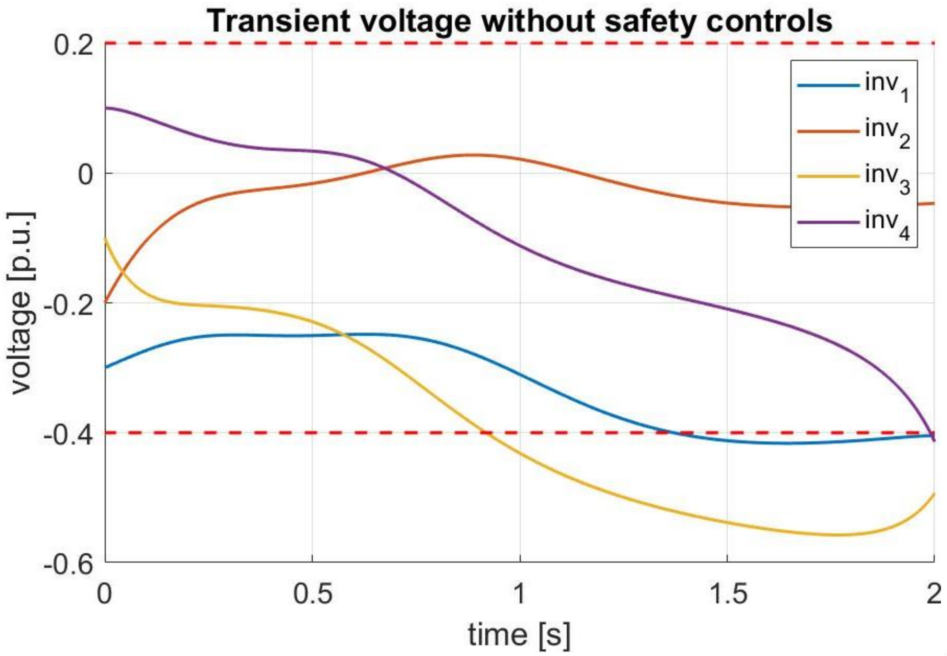


Source: EU MIGRATE Report

… need transformational change to achieve extreme high PE penetration (>75%)

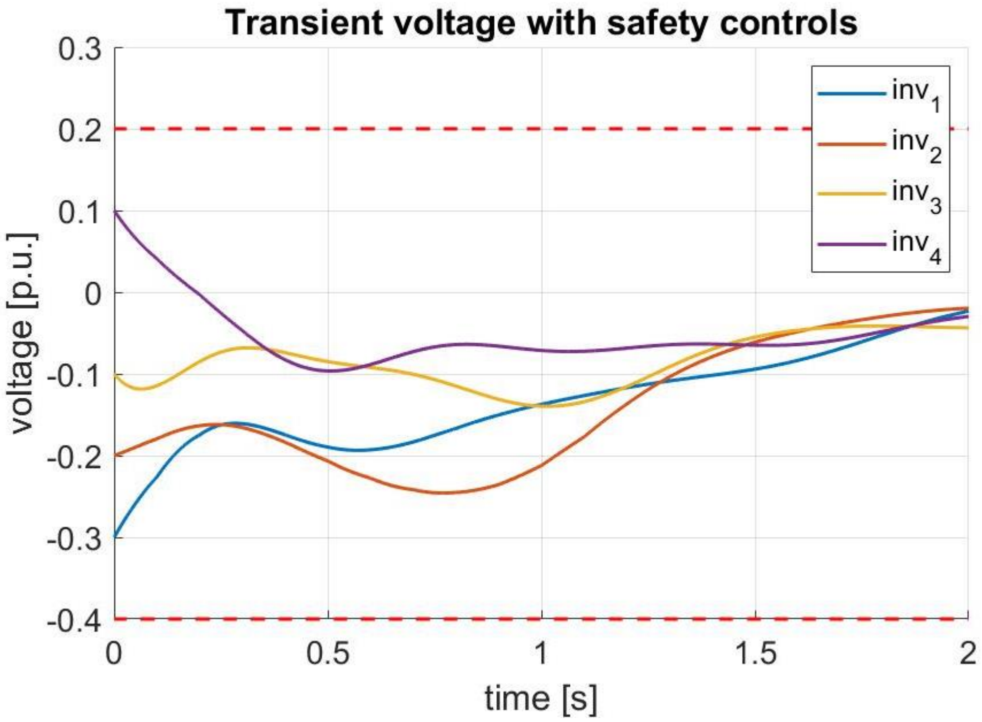
Problem: Local Transient Safety Constraints





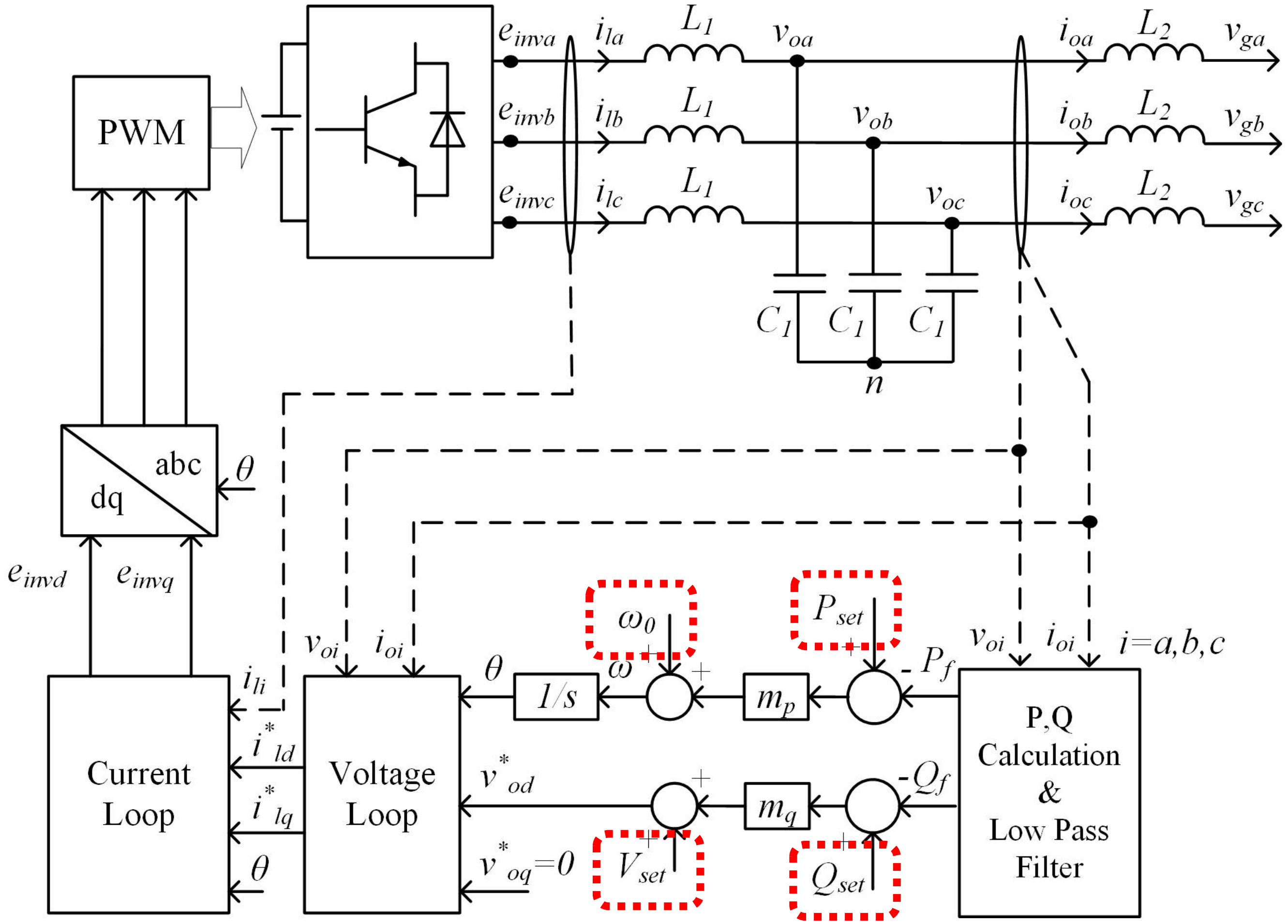
unsafe excursions in voltages

desired transient with safety controls

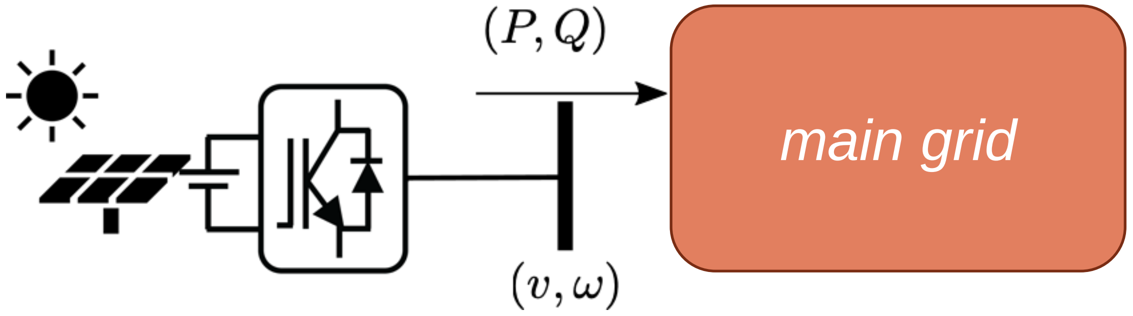


Local disturbances (e.g., solar fluctuations) cause unsafe excursions in voltages

Emerging Technologies: Grid-Forming Inverters



Multi-loop droop-control (P-ω, Q-V)

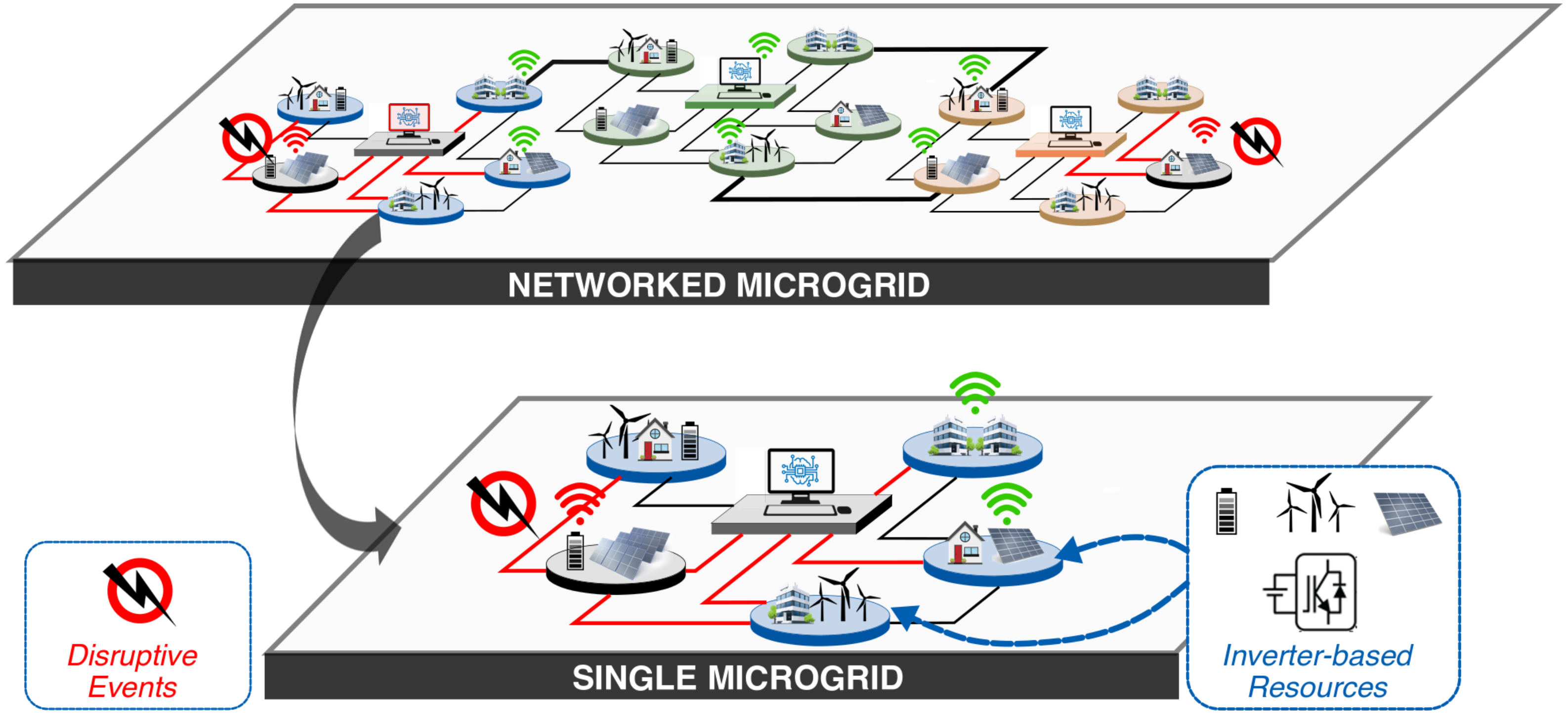


- Grid-forming inverters
  - Provides virtual inertia; acts as a voltage source; stable synchronization via inner control loops; black-start, and more …
- *Multi-loop droop-control* regulates voltage and frequency by controlling power (P,Q)

$$\omega_{\text{set}} = \omega^*_{\text{set}} - \lambda_p (P - P_{\text{set}}) \qquad \text{(P-}\omega\text{ droop)}$$

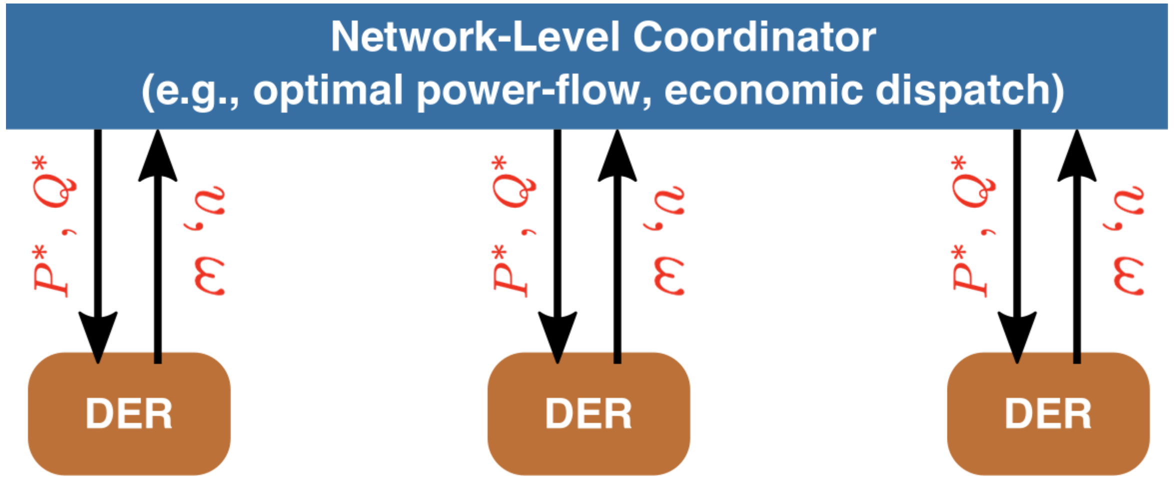
$$v_{\text{set}} = v^*_{\text{set}} - \lambda_q (Q - Q_{\text{set}}) \qquad \text{(Q-V droop)}$$

Hierarchical and Distributed Framework



- Hierarchical and distributed operations to coordinate many distributed energy resources (DERs) over the network
- Individual resources (e.g., inverters) received control set-points to track

Example: Optimal Power-Flow
- DERs receive set-points; in turn regulates voltage and frequency

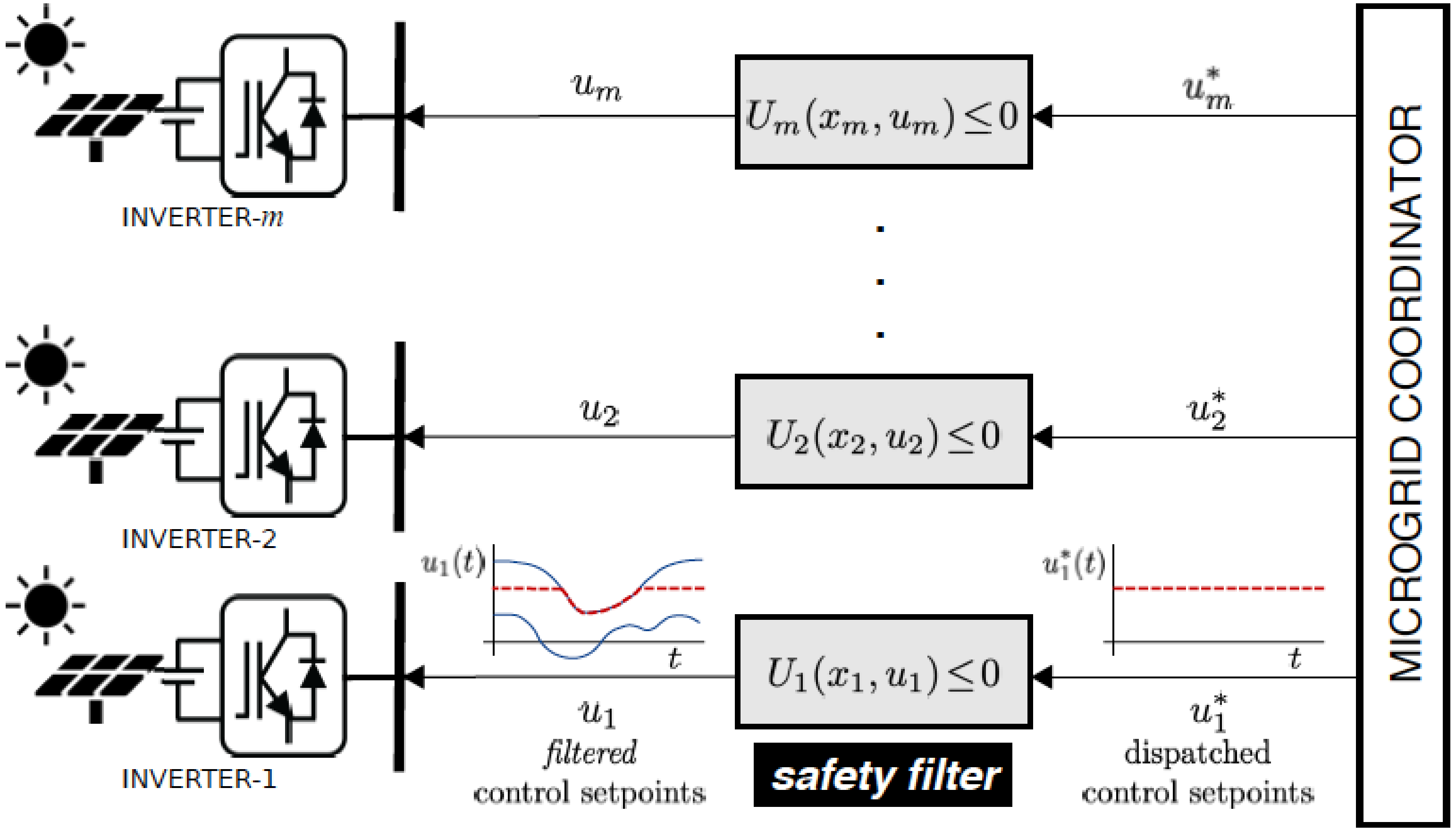


Multi-timescales Problem

State-of-the-art operational practices in inverter-based microgrids lack the spatiotemporal granularity required to proactively prevent transient safety and stability violations which are often local and fast-evolving in nature.

Safety Filter: The Concept

Decouple network-level objectives from local transient safety constraints



- Safety filters are deployed locally at the inverter terminals, and act as gatekeepers for allowable (safe) set-points
  - *State-inclusive bounds on the allowable control set-points*
- In a robust design, guarantees transient safety constraint satisfaction under bounded uncertainties in the network

Kundu and Kalsi, "Transient Safety Filter Design for Grid-Forming Inverters," ACC 2020.

Droop-controlled inverter dynamics

Local dynamics

$$\dot{\theta}_i = \omega_i$$
$$\tau_i \dot{\omega}_i = -\omega_i + \lambda_i^p \left(P_i^0 + u_i^p - P_i \right)$$
$$\tau_i \dot{v}_i = v_i^0 - v_i + \lambda_i^q \left(Q_i^0 + u_i^q - Q_i \right)$$

$$P_i = v_i \sum_{k \in N_i} v_k \; P_{i,k}(\theta_i, \theta_k)$$
$$Q_i = v_i \sum_{k \in N_i} v_k \; Q_{i,k}(\theta_i, \theta_k)$$

Neighbor interactions

Controls u_i^p and u_i^q

Safe sets $S_v = [\underline{v}, \overline{v}]$, and $S_\omega = [\underline{\omega}, \overline{\omega}]$.

Local measurements θ_i, ω_i, v_i are known, P_i and Q_i are unknown.

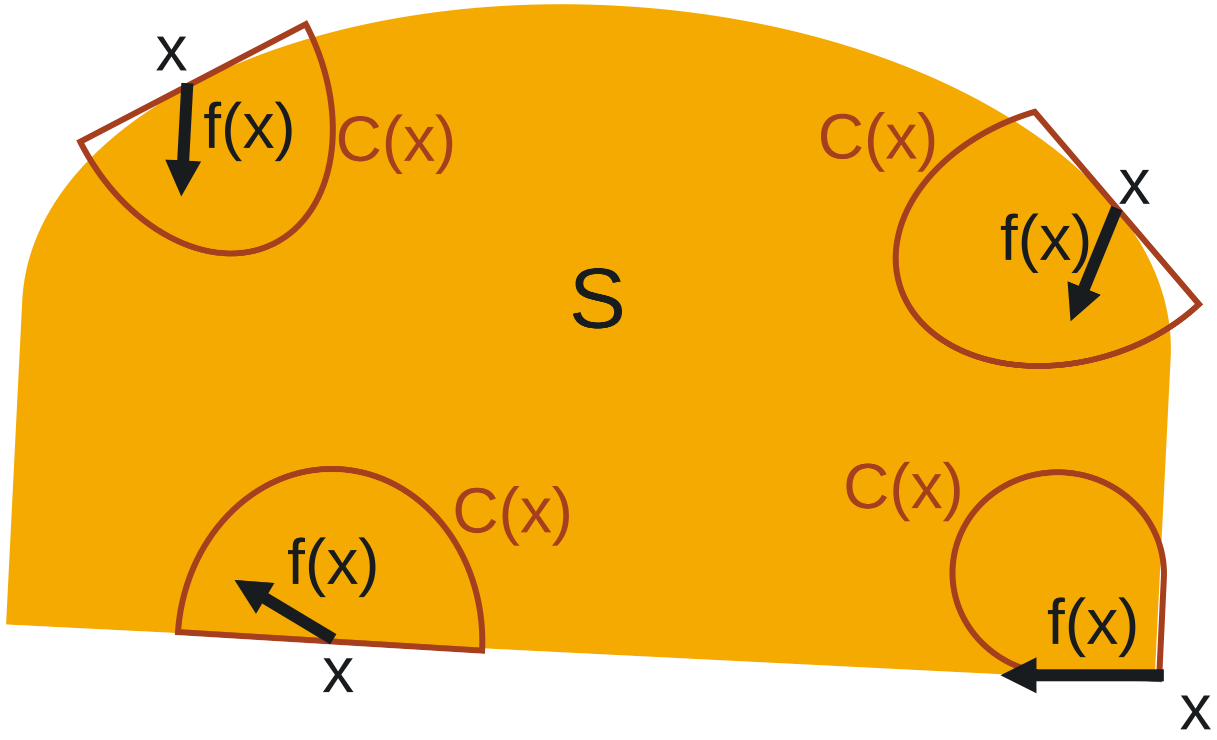
Problem How to maintain $v \in S_v$ and $\omega \in S_\omega$ during transients?

Invariant sets

S is **invariant** by $\dot{x} = f(x)$ if for all $x(0) \in S$, $\mathrm{x(t)} \in S$ for $\mathrm{t} \geq 0$.

Nagumo theorem

A closed set S is invariant by $\dot{x} = f(x)$ if and only if for all $x \in S$, $f(x) \in C(x)$, the Bouligand tangent cone to S at x.



S is **robust control invariant** by $\dot{x} = f(x, u, w)$ if there exists a control law $u(t)$ such that for all $\mathrm{x}(0) \in S$ and all $\mathrm{w} \in W$, $\mathrm{x(t)} \in S$ for all $\mathrm{t} \geq 0$.

Blanchini, "Set invariance in control," Automatica 1999.

Upper and lower invariance of safe sets

$S = [\underline{s}, \overline{s}]$ is **upper invariant** (resp. **lower invariant**) for U by $\dot{x} = f(x, u, w)$ if for all $u \in U$, $w \in W$ and all $x(0) \in S$, $x(t) \leq \overline{s}$ (resp. $x(t) \geq \underline{s}$) for all $t \geq 0$.

If $\dot{x} = g(x, w) + \lambda u$ with $\lambda > 0$, we define a **minimal lower control** $\underline{u}$ and a **maximal upper control** $\overline{u}$ such that

$$\underline{u} = min\{u_{low} \in U : S \text{ is lower invariant for all } u \geq u_{low}\},$$

$$\overline{u} = max\{u_{up} \in U : S \text{ is upper invariant for all } u \leq u_{up}\}.$$

$U = [\underline{u}, \overline{u}]$ is the maximal interval of **safety admissible** controls making S robust control invariant.

Extremal upper and lower controls

$$\overline{u_i^p} = \max_{\theta_k, v_k} \{u_i^p : \dot{\omega}_i \leq 0, \omega_i = \overline{\omega}\} = \max_{\theta_k, v_k} \frac{1}{\lambda_i^p} \overline{\omega_i} + P_i - P_i^0$$

$$\underline{u_i^p} = \min_{\theta_k, v_k} \{u_i^p : \dot{\omega}_i \geq 0, \omega_i = \underline{\omega}\} = \min_{\theta_k, v_k} \frac{1}{\lambda_i^p} \underline{\omega_i} + P_i - P_i^0$$

s.t. $\theta_k \in S_\theta, v_k \in S_v$ for all $k \in N_i$, and $S_\theta = [\underline{\theta}, \overline{\theta}]$.

Only P_i depends on θ_k, v_k. Thus, let

$$P_i^{max} = \max_{\theta_k, v_k} \{P_i : \omega_i = \overline{\omega}\},$$

$$P_i^{min} = \min_{\theta_k, v_k} \{P_i : \omega_i = \underline{\omega}\}.$$

So that $\overline{u_i^p} = \frac{1}{\lambda_i^p} \overline{\omega} + P_i^{max} - P_i^0$ and $\underline{u_i^p} = \frac{1}{\lambda_i^p} \underline{\omega} + P_i^{min} - P_i^0$.

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Maximal droop for constant invariance

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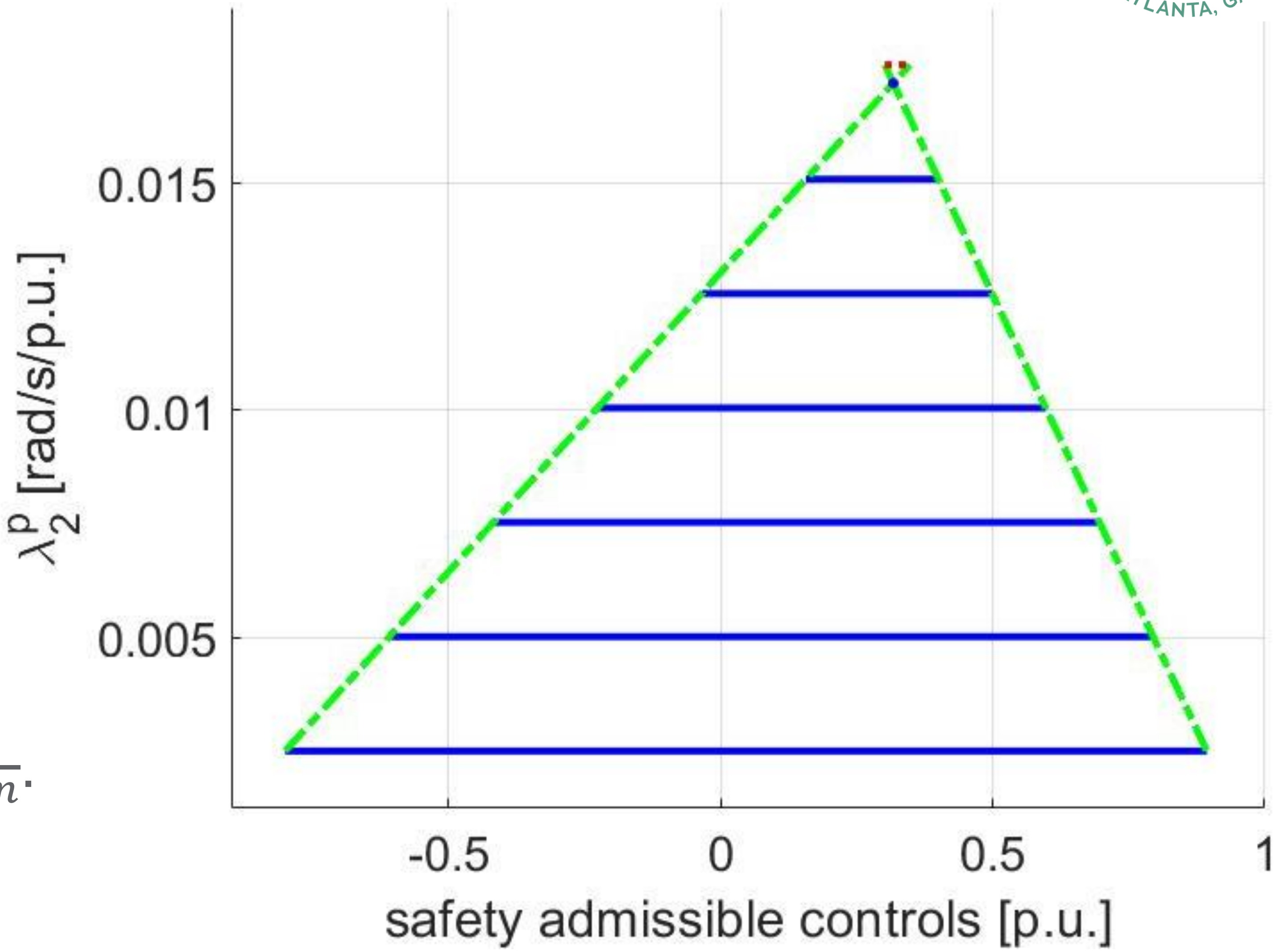
Safety admissible controls $U_i^p = \left[\underline{u}_i^p, \overline{u}_i^p\right]$ shrinks as λ_i^p increases.

$$\lambda_i^{p*} = max\left\{\lambda_i^p : \underline{u}_i^p(\lambda_i^p) \leq \overline{u}_i^p(\lambda_i^p)\right\}.$$

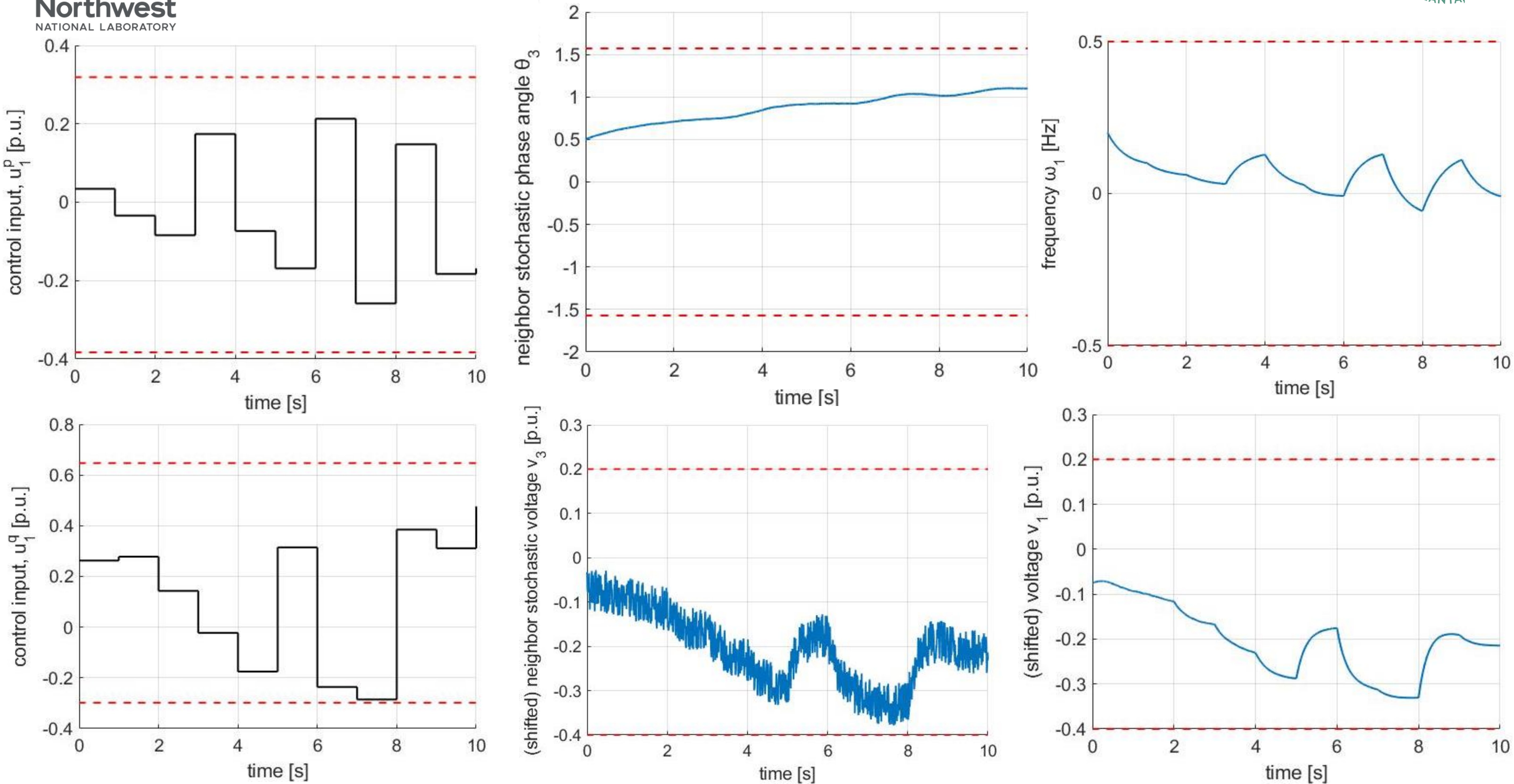


$\underline{u}_i^p(\lambda_i^{p*}) = \overline{u}_i^p(\lambda_i^{p*})$ yields $\lambda_i^{p*} = \dfrac{\overline{\omega} - \underline{\omega}}{P_i^{max} - P_i^{min}}$.

Sum-of-Squares algorithms to calculate P_i^{min} P_i^{max}.

Language & SDP solver	MATLAB SeDuMi	Julia SDPA	Julia Mosek
Run-time for P_i^{min}, P_i^{max}	4295s ~1h12	343s ~ 6min	33s

Microgrid simulation



Conclusion

Problem: how to prevent transient safety violations in inverter-based microgrids?

Solution: we relied on Nagumo’s theorem to ensure robust control invariance of the frequency and voltage safe sets.

Thank you